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Parametric approximation of airfoil aerodynamic coefficients at high angles of attack

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Abstract

Three methods for estimating the lift and drag curves in the 360° angle of attack (α) range with harmonic approximation functions were analyzed in the present work. The first method assumes aerodynamic response of a flat plate, the second utilizes even sine and even cosine approximation functions, and the third method, also utilizing trigonometric functions, was developed with the scope on stall-regulated turbines. The method of the even sine and cosine functions was further developed in the present work by using two independent harmonic approximations in the positive and negative α regions for the estimation of the lift and drag coefficients, and by using four independent harmonic approximations for the estimation of the moment coefficient. Further, it was determined that between α equal to 160° and -160° , the aerodynamic coefficients may be obtained with computationally inexpensive steady two-dimensional Computational Fluid Dynamics (CFD) computations. This was done by a comparison of the results obtained with 2D steady CFD with 3D unsteady CFD. In the present work, reference aerodynamic coefficients were used directly in this α region. Reference aerodynamic coefficients were also used directly in the α region between -30° and 30° as in this region the data is either available or may be computed with 2D CFD. In between the aforementioned α regions, the present approximation method produced lift, drag, and moment coefficient curves satisfactorily close to the reference by using several data points to tune the model, that would otherwise be calculated with 3D CFD.

Keywords:

aerodynamic coefficients, lift, drag, moment, high angles of attack, separation, modelling, harmonic functions, even sine, even cosine

1 Introduction

Authors dealing with wind turbine blade stability at standstill conditions, e.g. Gaunaa and Larsen [1], Skrzypiński and Gaunaa [2], and Petersen et al. [3] indicate that the aerodynamic damping of blades at high angles of attack (α) is dependent on their aerodynamic characteristics. Often, those characteristics are assumed to be as of a flat plate due to the fact that at high values of α , airfoils effectively resemble flat plates as the flow already separates at the leading edge. According to [1,2], a more accurate representation of airfoil characteristics at high values of α may be necessary to tackle the problem of aerodynamic stability at standstill conditions. Bak and Timmer [4] describe the state of the art in characterizing airfoils at high values of α .

This paper presents an attempt to approximate airfoil aerodynamic coefficients in the whole $-360^\circ - \alpha$ range with an engineering model and as limited amount of data points necessary to tune the model as possible. In the present work, those data points were extracted from the reference aerodynamic data. In case of future approximations of aerodynamic coefficients, 3D Computational Fluid Dynamics (CFD) would be used to obtain such data points at high values of α .

2 Tools and Methods

At high α , it is often assumed that the aerodynamic response of the airfoil is identical to that of a flat plate:

$$C_l = 2 \sin(\alpha) \cos(\alpha) \quad (1)$$

$$C_d = C_{d \max} \sin^2(\alpha) \quad (2)$$

$$C_m = -\sin(\alpha) / 4 \quad (3)$$

Alternative approach is presented by Apostolyuk [5] who approximates C_L and C_D of a modern jet fighter, based on experimental data by Hoffler et al. [6], with the harmonic even sine and even cosine approximation functions, respectively:

$$C_L = l_0 + \sum_{i=1}^n l_i \sin(2i \alpha) \quad (4)$$

$$C_D = d_0 + \sum_{i=1}^n d_i \cos(2i \alpha) \quad (5)$$

where n is the number of harmonic terms used to approximate either C_L or C_D , and l_i and d_i are the independent parameters for C_L and C_D , respectively, that are to be determined based on the best fit of either experimental or computational data. Apostolyuk concludes that both even functions deliver good approximation already with one or two harmonic terms, thus requiring two or three independent parameters l_i and d_i to be identified. The parameters are identified by solving the following system of equations, here for three independent parameters l_i and d_i , where three C_L and C_D values are known:

$$C_{Lj} = l_0 + l_1 \sin(2 \alpha) + l_2 \sin(4 \alpha) \quad (6) \\ j = 1, 2, 3$$

$$C_{Dj} = d_0 + d_1 \cos(2 \alpha) + d_2 \cos(4 \alpha) \quad (7) \\ j = 1, 2, 3$$

The solution of Equations (6) and (7) requires three independent measurements or computations of C_L and C_D at different α values. Present research indicated that, in certain cases, Moore-Penrose pseudoinverse of matrix or an alternative method may be necessary to solve the systems as the coefficient matrices may be close to singular. Further, Equations (6) and (7) applied to a number of data points may, in few cases, return a curve of shape significantly different from what may be expected. Then, a relaxation should be applied in which the data points are marginally moved

until a solution is found. Apostolyuk also points out that the benefit of his approach comes from the ability to identify the parameters l_i and d_i in a way most suitable for a given problem. For example, if greater accuracy is required at certain α region, the C_L and C_D values to identify l_i and b_i in Equations (6) and (7), respectively, may be obtained at that α region.

Another approach to estimate the aerodynamic coefficients at high values of α , with the scope on stall-regulated wind turbines, is presented by Viterna and Corrigan [7] who propose the following equations:

$$C_l = A_1 \sin(2 \alpha) + A_2 \frac{\cos^2(\alpha)}{\sin(\alpha)} \quad (8)$$

$$C_d = B_1 \sin^2(\alpha) + B_2 \cos(\alpha) \quad (9)$$

where

$$A_1 = 0.5 C_{d \max} \quad (10)$$

$$A_2 = (C_{l_s} - C_{d \max} \sin(\alpha_s) \cos(\alpha_s)) \frac{\sin(\alpha_s)}{\cos^2(\alpha_s)} \quad (11)$$

$$B_1 = C_{d \max} \quad (12)$$

$$B_2 = C_{d_s} - \frac{C_{d \max} \sin^2(\alpha_s)}{\cos(\alpha_s)} \quad (13)$$

The parameter α_s is the stall α at which the C_l curve peaks. The parameters C_{l_s} and C_{d_s} are the respective values of C_l and C_d . The method requires determination of the aerodynamic parameters at stall which may be obtained by computationally inexpensive 2D CFD, and determination of $C_{d \max}$ which is found at α equal to 90° independent of the profile, and therefore requires only a single 3D CFD computation.

In the present work, the aforementioned methods were verified against the 360° wind tunnel measurements of the DU96-W-180 airfoil carried out in the wind tunnel of the Delft University of Technology by Timmer and van Rooij [8,9]. The best method was chosen and modified in order to produce a satisfactory approximation of airfoil aerodynamic coefficients in the whole α range with the lowest possible number of data points necessary to tune the model.

In addition, a method for the approximation of the moment coefficient (C_m) similar to those proposed by Apostolyuk for the lift (C_l) and drag (C_d) coefficients was found.

Additionally, a range of α in the vicinity of 180° was found at which the aerodynamic coefficients may be computed with computationally inexpensive steady 2D CFD. This was done by a comparison of the results obtained with 2D steady CFD with the results obtained with unsteady 3D CFD.

Note that obtaining the aerodynamic characteristics in the α range approximately between -30° and 30° is not as difficult as obtaining this data at high values of α . This is because at low α , this data may be computed with 2D CFD or is readily available from wind tunnel measurements. Therefore, this α range is outside the scope of the present work. On the other hand, accurate prediction of airfoil aerodynamic characteristics at high values of α by means of 2D CFD or wind tunnel measurements proves very difficult. The flow at high α is heavily separated and three-dimensional. Therefore, it may not be accurately represented by 2D CFD. On the other hand, wind tunnel measurements at high α are difficult due to tunnel limitations such as blockage, although such measurements are carried out [8,9]. CFD computations capable of resolving the physics of flows at high α are 3D and computationally expensive. This increases the need for an engineering model capable of delivering reliable airfoil characteristics at high α by using a relatively low number of reference data points obtained by 3D CFD or wind tunnel measurements.

3 Results

3.1 Approximation of the lift and drag coefficients

The level of accuracy of the three methods mentioned in the preceding section is visualized in Figure 1 and Figure 2 by comparing the C_l and C_d curves of the DU96-W-180 airfoil approximated in the whole α range with the coefficients measured by Timmer and van Rooij [5,6].

The method by Viterna and Corrigan [4] estimated the C_l curve well only in the α range between stall and 120° . The two other methods worked well as low fidelity approximations in the whole α range. The methods seem incapable of representing different maximum C_l values in the negative and positive α regions. In addition, they do not seem to capture the peculiarities of the curve in or close to the attached flow region as well as around 180° . All three approximations of the C_d curve presented in Figure 2 produced effectively the same result, working well as low fidelity approximations in the whole α range but not predicting the maximum C_d in the positive α region. Note that the α region approximately between -30° and 30° , which includes the attached flow region, is typically computed with 2D CFD or is available from wind tunnel measurements. Therefore, it was outside the scope of present work.

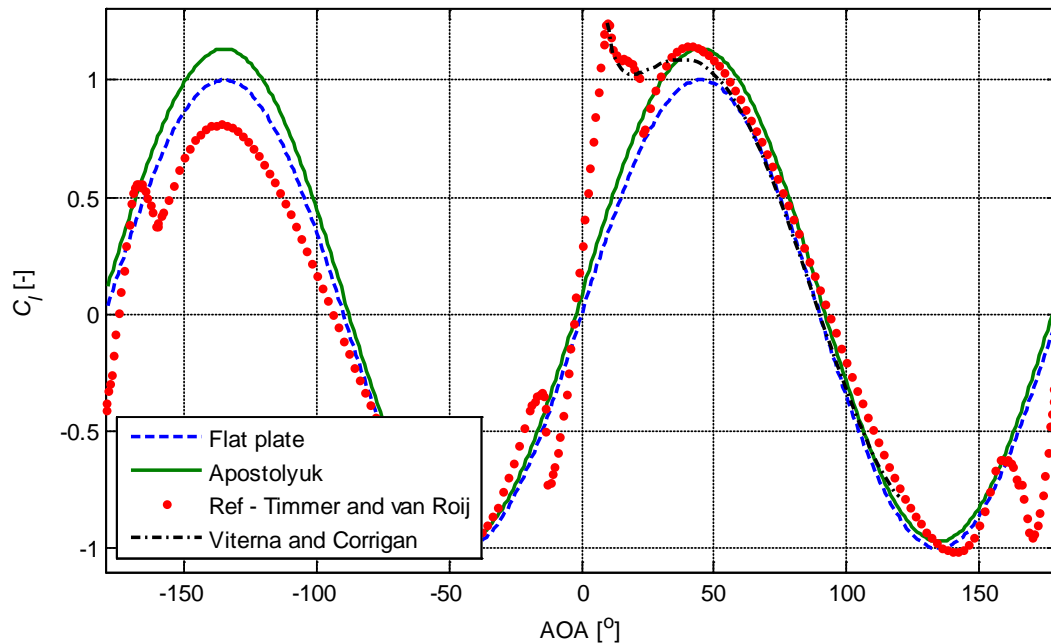


Figure 1: Three different methods for predicting C_l in deep stall are compared with the reference measurement data by Timmer and van Rooij [8,9].

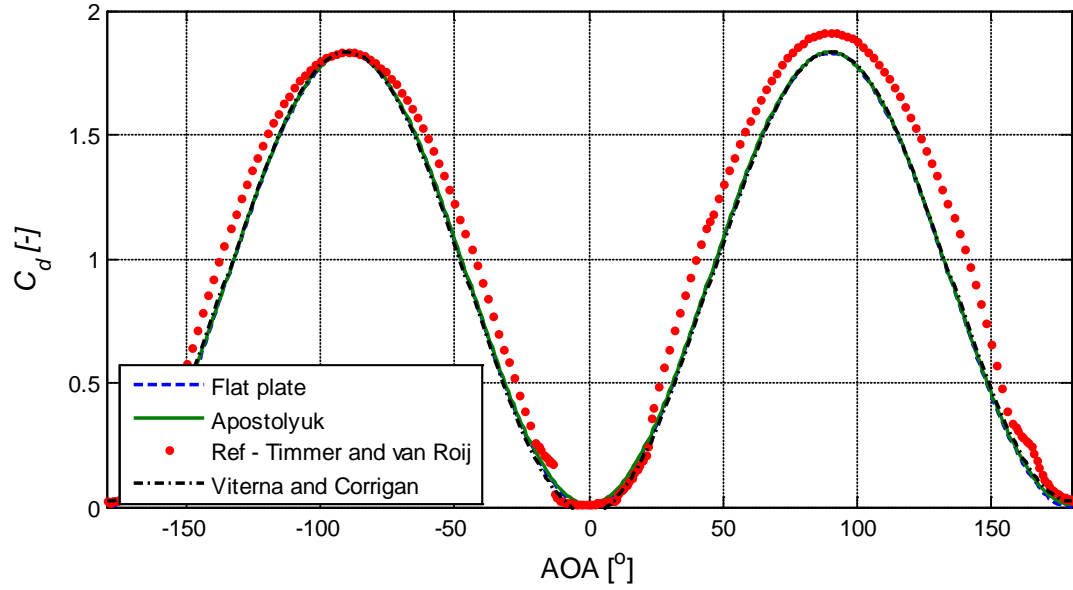


Figure 2: Three different methods for predicting C_d in the whole α range are compared with the reference measurement data by Timmer and van Rooij [8,9]

In the present work, it was also shown that the aerodynamic characteristics at values of α in the vicinity of 180° may be computed with 2D CFD. This is because the flow in this α region is partly attached. In other words, the airfoil experiencing reversed flow effectively appears as an airfoil with a sharp leading edge and a blunt trailing edge. Figure 3 presents a comparison of 2D steady CFD computations carried out at α between 150° and -150° with

3D unsteady Detached Eddy Simulation (DES) CFD computations carried out at α equal to 163° , 169° and 180° . All the computations were carried out at $Re=12 \cdot 10^6$ on the FFA-W3-241 airfoil, developed at the Aeronautical Institute of Sweden [10]. The simulations were carried out in EllipSys2D and EllipSys3D Navier-Stokes solvers, developed by Michelsen [11, 12] and Sørensen [13] at DTU Wind Energy.

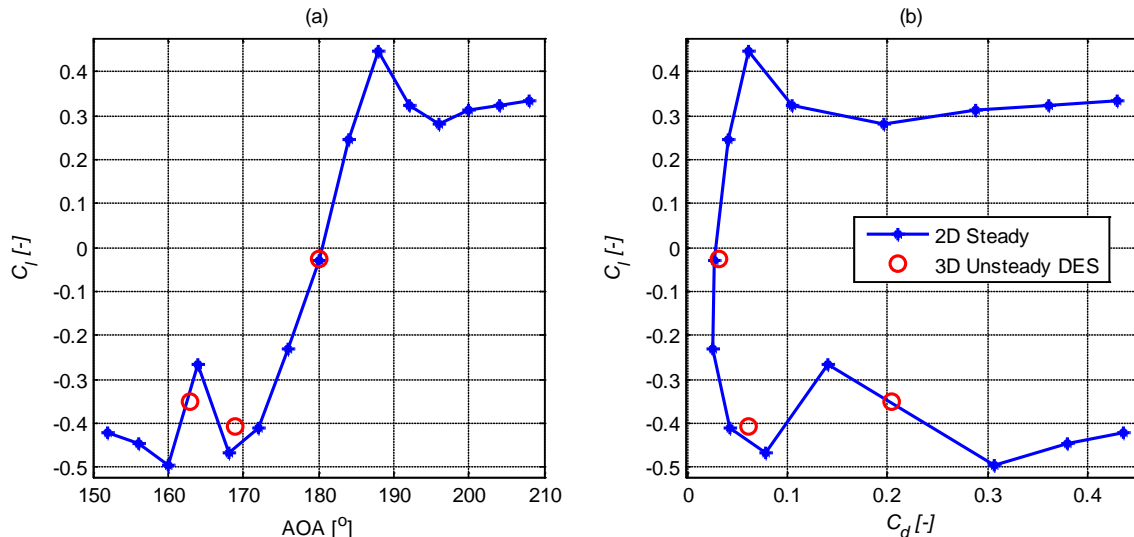


Figure 3: Comparison of 2D steady CFD computations carried out at α between 150° and -150° with 3D unsteady Detached Eddy Simulation computations carried out at α equal to 163° , 169° and 180° . All the computations were carried out at $Re=12 \cdot 10^6$ on the FFA-W3-241 airfoil.

The comparison was satisfactory, as the C_l and C_d values computed with 3D CFD were relatively close to the respective curves obtained by means of 2D computations. Because of that, in the present work the reference measurement data was used directly not only at α between -30° and 30° but also between 160° and -160° . It is also recommended that those α regions are computed using 2D CFD when 360° aerodynamic characteristics are approximated in the future using the present method.

The equations proposed by Apostolyuk were chosen for further investigation, with the even sine approximation for C_h and even cosine approximation for C_d . The most effective approach to capture the different maxima in the positive and negative α regions of both C_l and C_d curves was to find independent approximations for the curves in both regions, and blend them with the characteristics used directly at α between -30° and 30° as well as between 160° and -160° . The results of such approximations for C_l and C_d are presented in Figure 4 and Figure 5, respectively.

A satisfactory estimation of the C_l curve was obtained with four data points used to tune the model on each side of the y axis. At the right side of the y axis, those points were at α equal to: 30° , 45° , 105° and 135° . On the left side of the y axis, the points were placed symmetrically. If points at the same values of α

were used to approximate a new airfoil, three points at each side of the y axis would need to be computed using 3D CFD whereas at α equal to -30° and 30° , 2D CFD would likely be sufficient.

In order to approximate the C_d curve, three points at each side of the y axis were used: 30° , 45° and 105° .

3.2 Approximation of the moment coefficient

The estimation of the C_l and C_d curves differed from the estimation of the C_m curve as the C_m curve was estimated by a total of four even cosine harmonic functions – two even cosine functions in the positive α region and two in the negative. The following α regions were approximated with independent even cosine functions: $[-163^\circ : -135^\circ]$, $[-135^\circ : -30^\circ]$, and symmetrically in the positive α region. In the positive α region, the data points used to tune the model were at α equal to: 30° , 135° and 160° where only the point at 135° would be computed with 3D CFD in case of a new approximation. Note that C_m is not approximated in the original work of Apostolyuk [5] whereas its approximation may be relevant in the context of wind turbine aerodynamic stability. The results are presented in Figure 6.

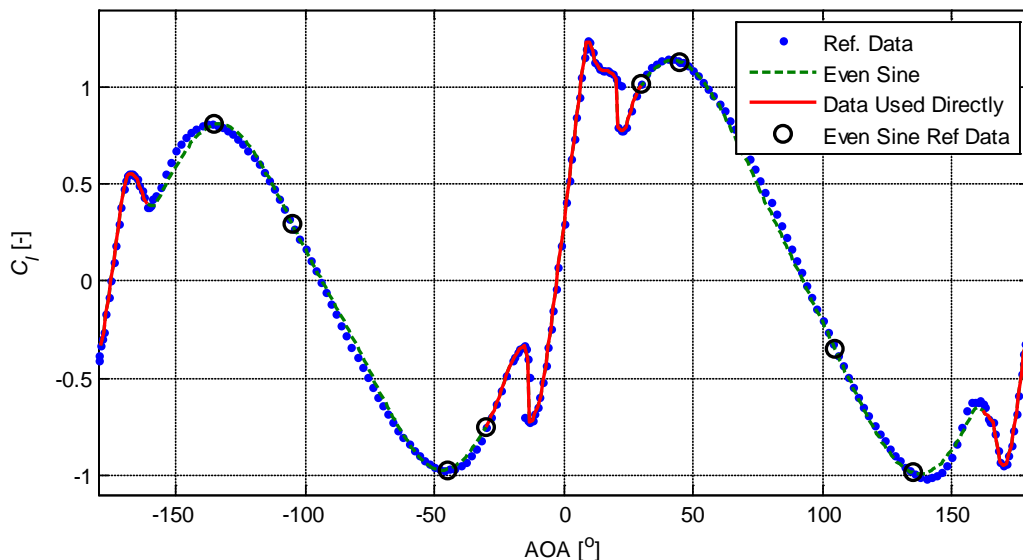


Figure 4: Even sine approximation of C_l , blended with the reference data by Timmer and van Rooij [8,9] for the values of α between -30° and 30° as well as between 160° and -160° ; 'Even Sine Ref Data' shows the points used for tuning the model

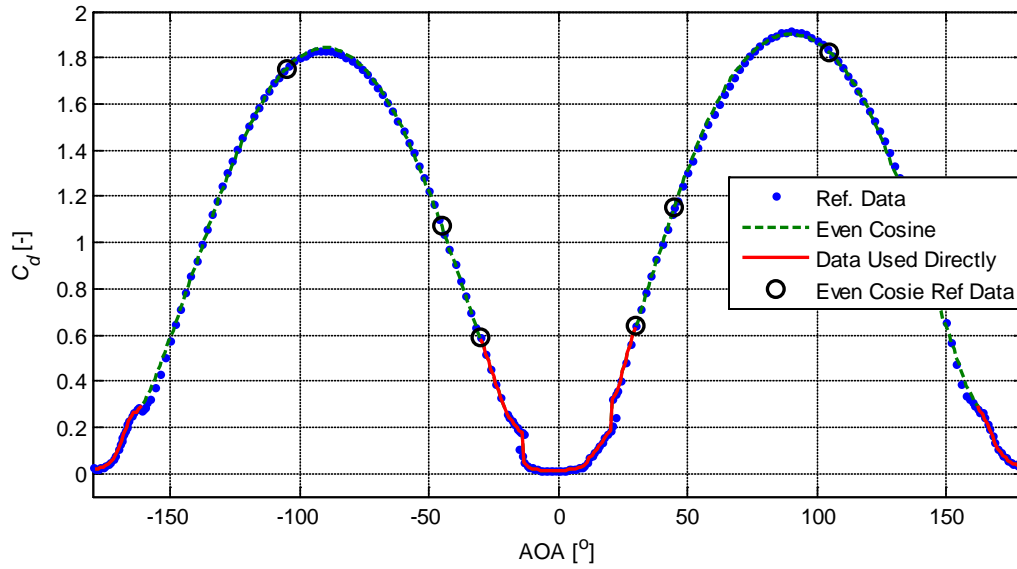


Figure 5: Even cosine approximation of C_d , blended with the reference data by Timmer and van Rooij [8,9] to the values of α between -30° and 30° as well as between 160° and -160° ; 'Even Cosine Ref Data' shows the points used for tuning the model

The present method produced satisfactory results for the C_l , C_d and C_m curves in the whole α range by using five data points to tune the model on each side of the y axis. In case of a new approximation, three points at α equal to 45° , 105° and 135° would need to be computed using 3D CFD while the points at 30° and 160°

would likely be obtained using 2D CFD. Note that in principle, CFD computations of airfoils may always be substituted by wind tunnel measurements, both characterized by certain limitations.

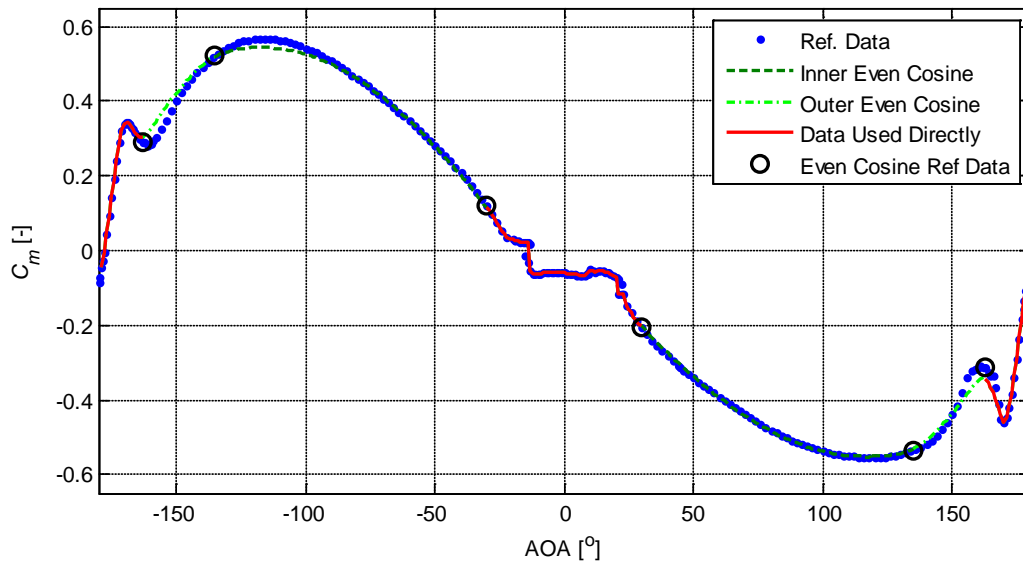


Figure 6: Approximation of C_m , blended with the reference data by Timmer and van Rooij [8,9] in the α region between -30° and 30° as well as between 160° and -160° ; 'Even Cosine Ref Data' shows the points used for tuning the model

3.3 Test of the present method

In order to approximate the aerodynamic coefficients of the DU96-W-180 airfoil described in the previous section, the data points used for tuning the model were chosen in order to achieve the best fit to the reference data. The question that naturally arose was whether the choice of the same data points brings satisfactory results in case of other airfoils. Only then, the present method may be used in a

broader context. This section presents such a test carried out on the reference C_l and C_d curves of the Risø-B1-18, which were obtained at Risø by means of wind tunnel measurements by Fuglsang et al. [14] approximately in the α range from -5° to 27° and several interpolated EllipSys 3D CFD computations in the remaining α range.

The results are presented in Figure 7 and Figure 8.

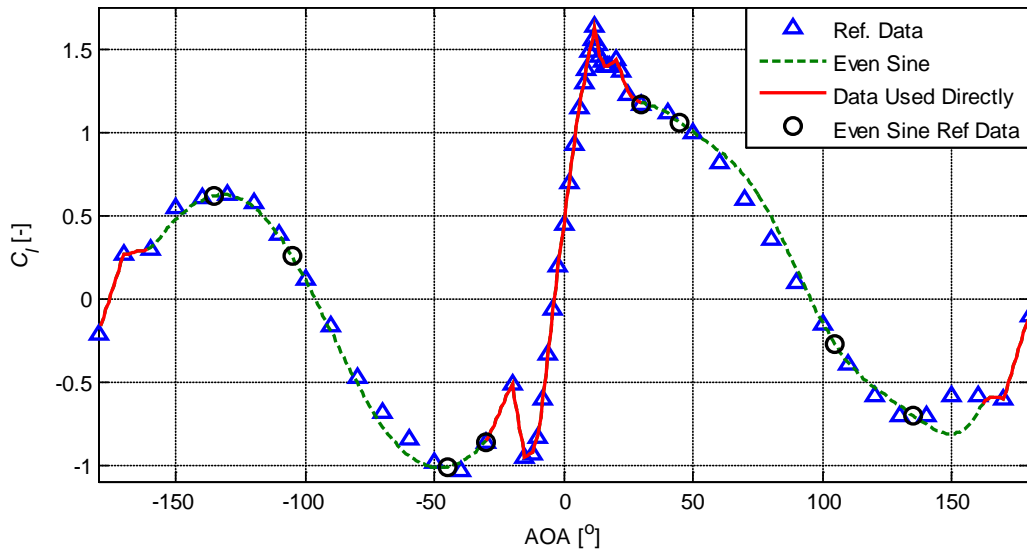


Figure 7: Approximation of the C_l curve of the Risø-B1-18 airfoil blended with the reference data [14] in the α region between -30° and 30° as well as between 160° and -160° ; 'Even Sine Ref Data' shows the points used for tuning the model

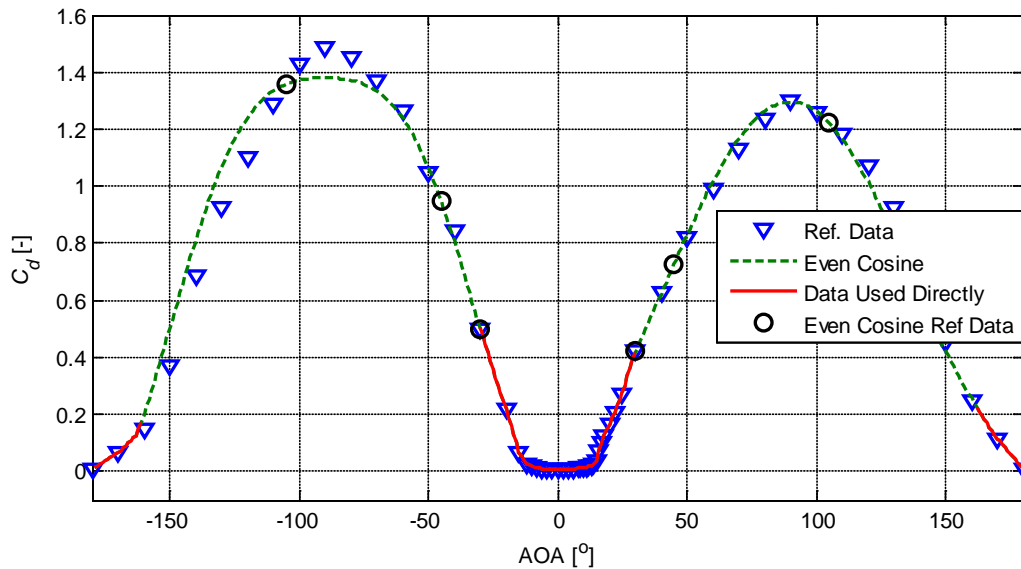


Figure 8: Approximation of the C_d curve of the Risø-B1-18 airfoil blended with the reference data [14] in the α region between -30° and 30° as well as between 160° and -160° ; 'Even Cosine Ref Data' shows the points used for tuning the model

Generally, the approximated curves are satisfactory although not as close to the reference as the approximation of the DU96-W-180 coefficients presented in the previous sections. However, the present method, although it may need further validation and tuning, potentially may serve as a tool for estimation of airfoils' aerodynamic coefficients at high values of α .

Note that C_d measured on a 2D airfoil at high α may be different from what would be measured on a corresponding blade. This phenomenon is investigated by Sørensen and Michelsen [15]. Assuming this difference, the 2D measurements from wind tunnels should not be applied directly in the context of wind turbines. Instead, one could carry out few computations or measurements at high values of α on a 3D blade, and then extract the coefficients using the present model. Zahle et al. [16] derive airfoil data for standstill conditions from 3D DES CFD computations on the DTU 10 MW Reference Wind Turbine [17].

4 Conclusions

Three methods for estimating the C_l and C_d curves in the whole α range by using harmonic equations were analyzed in the present work. The first assumes aerodynamic response of a flat plate. The second utilizes even sine and even cosine functions, proposed by Apostolyuk [5] for the estimation of the aerodynamic coefficients of modern jet fighter planes. The third method by Viterna and Corrigan [7], developed with the scope on stall-regulated turbines, also utilizes trigonometric functions. The methods were validated using aerodynamic coefficients of the DU96-W-180 airfoil [8, 9]. The method of Apostolyuk was further developed by using two independent harmonic approximations in the positive and negative α regions for the estimation of C_l and C_d , by using four independent harmonic approximations for the estimation of C_m . Original data was used directly in the α region between -30° and 30° as well as between 160° and -160° . In order to use original data between 160° and -160° and thereby assume that in the future this α region would be computed with 2D CFD, a comparison of the results obtained with 2D and 3D CFD in the aforementioned α region was carried out. Those computations were carried out on the FFA-W3-241 airfoil using EllipSys2D and EllipSys3D Navier-Stokes solvers [11,12,13].

The method produced satisfactory results for the C_l , C_d and C_m curves in the whole α range by using five data points to tune the model on each side of the y axis. In case of a new approximation, three points at α equal to 45° , 105° and 135° would need to be computed using 3D CFD while the points at 30° and 160° could be obtained using 2D CFD. In the end, the method was validated on the reference aerodynamic data concerning the Risø-B1-18 airfoil [14].

5 Future work

Note that the present work relies on two sets of data. It is therefore a first step in the creation of a generic and reliable engineering model where further validation, tuning and development should be carried out on additional data sets.

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